Essential grammar we have learned: first just math

Latitude, longitude, altitude

Zonal, meridional, vertical

Northward vs. northerly; eastward vs. westerly; poleward vs. equatorward; cyclonic

upward, altitude, pressure level (know Earth's atmosphere layers, z & p depth values)

troposphere, stratosphere, planetary boundary layer (PBL)

Cartesian: x,y,z coords **i,j,k** unit vectors u,v,w components of vector wind **V**

*tangent plane* to the spherical Earth, locally accurate, *orthogonal*

a scalar vs. a vector

a vector has 2 *properties* (in 2D or 3D): *magnitude* and *direction*

Addition (graphical), two kinds of multiplication: dot product, cross product

defined for an individual vector; repeated at every location for a vector *field*

scalar *field* vs. vector *field* -- know your MKS Units on all these quantities

position, velocity, acceleration, force, per-unit-mass, energy, flux of anything

*functions* apply over a *domain* (coordinates or arguments or inputs)

their *value* (output) spans some *range* of outcomes

in other words, a function is a *mapping from domain to range*

graphically: a function is a *curve* (1D domain), *surface* (2D), or *field* (3D, 4D)

ex: T(x,y,z,t) is *temperature everywhere forever,* **V500**(x,y) is *hor. wind @500*

Derivatives of a function:

first: *slope* (on 1D domain) is a scalar; *gradient* (on 2D, 3D domain) is **vector field**

second: *curvature* (on any dimension of domain), a **scalar field**

second derivative is a *scalar operator*: flips sign, for pure *sinuoids* (waves)

equal to *divergence of gradient* on 2D or 3D domain

Del operator: a ***vector******operator***(*nabla* symbol), behaves just like a vector except that things to the left of it are *multiplied*, while things on the right of it are *operated on*.

that is, Del does not *commute* like a regular vector field.

\* know the *gradient* of a scalar function *of space*: result is a **vector** **field**

like *temperature gradient*  where T(x,y,z, maybe t) is a scalar function

\* know "del dot" a vector field

vergence (divergence, convergence if <0) div(**V**) = "del dot **V" =**

convergence of a flux is the local tendency due to transport

advection is equal (= transport tendency), because mass cont.

\* know "del squared": divergence of gradient, second derivative in space

a measure of *curvature* of a curve, surface, or (abstractly) 3D field

differentiation emphasizes small scales: *edge finder* in image for example

random exchange (molecules) creates a *diffusive* *flux, down the gradient*

*diffusivity* is the coefficient relating flux to gradient

*convergence* of that flux is a *transport tendency* called *diffusive tendency*

called *viscous force* for diffusive momentum flux

advection **"minus V dot del**" or "minus V dot grad T": *transport from upwind*

note negative sign

Curl of vector field **V**,

Only in 3D! Right hand rule.

(vector *vorticity*, if **V** is a 3D velocity field)

we mostly use only its *vertical component,*  = vy - ux

(where subscripts indicate partial derivatives)

Curl of gradient vanishes precisely - why?

round-and-round vs. uphill-downhill are the 2 kinds of motion

natural coordinates help us see this sometimes

Scale of variation (m vs. km vs. 1000s of km; hours vs. days vs. months):

notice these are *logarithmic* distinctions, not just "size" (like 10m vs. 5m) Running average (smoothing) isolates *large scales* (larger than *filter scale*)

Deviations from that are *small scales*: (*subfilter* scales)

*anomaly* (deviation from time average)

*eddy* (deviation from space average)

*perturbation*: someone/something *perturbed* something

away from some *control* case

(to do an *experiment*, capable of isolating *cause and effect*)

*Partial derivatives* of a field or multivariate function f(x,y,z,t)

*Local* or *Eulerian* **∂**f/**∂**t, but also **∂**f/**∂**x, **∂**f/**∂**y, etc. *All other args held constant*

so it matters what *the whole argument list* is:**∂**f/**∂**t|x,y,z /=/ **∂**f/**∂**t|x,y,p !

careful! this is often just left to context. Trust definitions as fallback!

*Total* or *Lagrangian* Dq/Dt, following a moving parcel at position <xp(t), yp(t), zp(t)>

Know the difference from to the local derivative **∂**q/**∂**t (chain rule):

advection by **V** = <u,v,w> = d/dt <xp(t), yp(t), zp(t)> = >

because the *rate of change* of *coordinate position* IS *velocity*

Nondivergent vs. irrotational decomposition of a vector field

equivalently, rotational and divergent, "**components**" *of field decomposition*

different meaning than *vector* ***components***(along the unit vectors)

*\* streamline, streamfunction, streamwise*: *instantaneous* velocity, *threaded* tip to tail

*\* trajectory* (different from streamline): motion of a parcel through time

Integral relationships (opposite of derivative) for gradient, div, curl

Stokes' theorem (circulation), Gauss' theorem (for divergence)

vanishing of div(curl(**V**(x,y,z))) <--> vanishing of loop integral of gradient

ODEs and solutions (which come from *integrating* the equation)

*time tendency* is usually put on *left hand side (LHS)*

exponential solutions to df/dt = -bf

sinusoidal solutions to d2f/dt2 = -c2f

exp() with complex numbers combines both

linear solutions to df/dt = constant

need boundary or initial conditions (constant of integration) to fully solve DEs

stationary or steady-state solution: equilibrium or "balance"

dq/dt = A - B. Make steady-state assumption. Is it still a diff-eq? NO! A=B

PDEs and solutions: terms and concepts (for our applications)

*prognostic* vs. *diagnostic* (does it have a time derivative?)

boundary conditions, initial conditions

inverse of Laplacian (smoothing, the opposite of "edge finding"; reversed sign)

*Streamfunction* is inverse-Laplacian of *vorticity*

*PROGNOSTIC* EQUATIONS:

Contains a time derivative or *rate of change*, customarily on left-hand side (LHS)

terms on RHS are then called *partial tendencies*, time *tendencies*

*Governing* equation(s), *budgets*, with *partial tendencies* (tendency terms) on RHS

*Eulerian (local*) vs. *Lagrangian (total, following-the-flow*) derivatives

d/dt(something) = 0 + sources - sinks "conserved except for sources-sinks"

**∂**/**∂**t(something) = flux convergence + sources - sinks

**∂**/**∂**t(something) = advection + sources - sinks

*Conserved* *tracer* an important special case: *sources-sinks negligible*

*Balance* special case: *neglect a time derivative relative to other tendencies*

*hydrostatic* for w= equation (pressure = weight = g x mass of air above)

*geostrophic wind balance, gradient wind balance* in the horizontal

*geostrophic wind* **V**g, *thermal wind* (**V**T = upper-level **V**g minus lower-level **V**g)

fictional wind fields obeying balance exactly: no divergence, unchanging

*jet stream* (a momentum feature; sketch forces, p surface slope, thickness)

*cool core cyclone*: positive PV feature in upper troposphere

*warm core cyclone*: positive PV feature strongest at low levels

*Adjustment* (a fast or efficient process of restoration/maintenance of balance)

*Dynamics*: the physical study of flow (forces, etc.) and changes (prognostic)

*Kinematics*: the math basis set of 4 2D spatial gradients of 2D velocity field

vorticity, divergence, deformation (has 2 properties: *strength* and *axis angle*).

diffluence/confluence may or may not be associated with true divergence

some recipes:

shear = vorticity + deformation

line of convergence = convergence + deformation

Waves: terms and concepts

harmonic or sinusoidal functions

frequency, period, wavelength, wavenumber, amplitude, phase

phase velocity, group velocity

growing, decaying *amplitude* (in space or time)

growing, shrinking *scale* (expressed as wavenumber or wavelength)

so be careful with that word grow !

& careful with "high" vs. "great" vs. "big/large" values of a quantity

Physical concepts/words to know

Mass, density, mass fractions (specific \_\_, mixing ratio of \_\_, concentration of \_\_)

Conservation of mass (*continuity* of mass flux)

Flux of mass, multiply by specific \_\_ to get flux of \_\_ (\_\_ = momentum, moisture, etc.)

Conservation of stuff

TRANSPORT:

Flux of (stuff): what are the units? (Stuff) per second per square meter, in 3D

Flux *convergence* is the impact of the flux (*transport's* "drop-off" or "delivery")

*Advection*: the sense of it (upstream conditions coming at ya) and the math ( -**V.del** )

how are *advection* vs. *flux convergence* related?

(Answer: equal, because of mass continuity, as in homework).

*Diffusion* (convergence of a flux that is negatively proportional to the gradient).

PHYSICAL LAWS

*Equation of Motion* / Newton’s 2nd Law (F=ma)

Gravity force: vertical; it *defines* *vertical*, and thereby *horizontal*

Pressure-gradient force (PGF): Enforcer of continuity, in general

Gradient of *pressure-surface height* or *geopotential height* in p-coords.

*Coriolis force* (if still air on rotating Earth is ‘motionless’, this is very real)

f is *Coriolis parameter*, also equal to *planetary vorticity*

“*Inertial forces*” (*advection of momentum* by wind itself)

“*Friction*” (convergence of momentum flux by small-scale motions; *viscosity*)

Vorticity equations: d/dt(vorticity) = 0 + complications (*sources-sinks*)

Relative vorticity : eliminates PGF from momentum equations (in p coords)

Absolute vorticity a=(f+) moves v df/dy = vb term to LHS in d/dt(a) = 0 +...

mechanism: Coriolis force *torque* on air patch that moves south or north

Potential vorticity PV, eliminates a div(**V**) "*vortex* *stretching*" term from RHS

d/dt(PV) = 0 is our closest-to-true conservation law

only *diabatic* and *frictional* terms (source-sink) on RHS

Vortex interactions (e.g. for TC steering): 2D reasoning

1/r decay of “induced” rotational wind from vorticity element (point vortex)

Vtan  (1/r)

rel itself is advected by the “induced” flow from all other vortex points

a *point vortex model* of flow and its predictability

Sketch how this plays out for 2 vortices of same/opposite sign

*Rossby waves*: includes advection of planetary vor (or conservation of absolute vor)

from d/dt(a)=0 with  =df/dy <-- (total deriv. because f depends on y only)

Phase velocity c=U - k2 : westward relative to U, long waves faster

Group velocity cg=U +  k2: eastward relative to U, " " "

"*downstream development*" process

For stationary waves (c=0), cg = 2U

First Law of Thermodynamics (conservation of microscopic energy)

heat energy added to gas = change in internal energy + work done by gas (p dV/dt)

Per unit mass: Q = Cv dT/dt + p da/dt

*Ideal gas law*, an *equation of state* for air: pa = RT

Plug in: Q = Cp dT/dt - a dp/dt = Cp dT/dt - aw <-- in p-coords

where Cp = Cv + R

Mass continuity

Hydrostatic pressure (or mass) coordinate makes it especially clean

omega = w = dp/dt = is vertical velocity within this coordinate system

notice: *negative for upward motion*

but also "pressure change with time" on Earth's surface